**PRACTICE AND PROBLEM SOLVING**

15. **Navigation** A sailor on ship A measures the angle between ship B and the pier and finds that it is 39°. A sailor on ship B measures the angle between ship A and the pier and finds that it is 57°. What is the measure of the angle between ships A and B?

The measure of one of the acute angles in a right triangle is given. What is the measure of the other acute angle?

16. \( 76 \frac{1}{4}^\circ \) 

17. \( 2x^\circ \) 

18. \( 56.8^\circ \)

Find each angle measure.

19. \( m \angle XYZ \)

20. \( m \angle C \)

21. \( m \angle N \) and \( m \angle P \)

22. \( m \angle Q \) and \( m \angle S \)

23. **Multi-Step** The measures of the angles of a triangle are in the ratio 1 : 4 : 7. What are the measures of the angles? (Hint: Let \( x \), \( 4x \), and \( 7x \) represent the angle measures.)

24. Complete the proof of Corollary 4-2-2.
   Given: \( \triangle DEF \) with right \( \angle F \)
   Prove: \( \angle D \) and \( \angle E \) are complementary.

   **Proof:**
   
<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \triangle DEF ) with rt. ( \angle F )</td>
<td>1. a. ?</td>
</tr>
<tr>
<td>2. b. ?</td>
<td>2. Def. of rt. ( \angle )</td>
</tr>
<tr>
<td>3. ( m \angle D + m \angle E + m \angle F = 180^\circ )</td>
<td>3. c. ?</td>
</tr>
<tr>
<td>4. ( m \angle D + m \angle E + 90^\circ = 180^\circ )</td>
<td>4. d. ?</td>
</tr>
<tr>
<td>6. ( \angle D ) and ( \angle E ) are comp.</td>
<td>6. f. ?</td>
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</tbody>
</table>

25. Prove Corollary 4-2-3 using two different methods of proof.
   Given: \( \triangle ABC \) is equiangular.
   Prove: \( m \angle A = m \angle B = m \angle C = 60^\circ \)

26. **Multi-Step** The measure of one acute angle in a right triangle is \( 1 \frac{1}{4} \) times the measure of the other acute angle. What is the measure of the larger acute angle?

27. Write a two-column proof of the Third Angles Theorem.
41. What is the value of \( x \)?
   - A 19
   - B 52
   - C 57
   - D 71

42. Find the value of \( s \).
   - F 23
   - H 34
   - G 28
   - I 56

43. \( \angle A \) and \( \angle B \) are the remote interior angles of \( \angle BCD \) in \( \triangle ABC \). Which of these equations must be true?
   - A \( m \angle A - 180^\circ = m \angle B \)
   - C \( m \angle BCD = m \angle BCA - m \angle A \)
   - B \( m \angle A = 90^\circ - m \angle B \)
   - D \( m \angle B = m \angle BCD - m \angle A \)

44. **Extended Response** The measures of the angles in a triangle are in the ratio 2:3:4. Describe how to use algebra to find the measures of these angles. Then find the measure of each angle and classify the triangle.

**CHALLENGE AND EXTEND**

45. An exterior angle of a triangle measures 117°. Its remote interior angles measure \((2y^2 + 7)°\) and \((61 - y^2)°\). Find the value of \( y \).

46. Two parallel lines are intersected by a transversal. What type of triangle is formed by the intersection of the angle bisectors of two same-side interior angles? Explain. (Hint: Use geometry software or construct a diagram of the angle bisectors of two same-side interior angles.)

47. **Critical Thinking** Explain why an exterior angle of a triangle cannot be congruent to a remote interior angle.

48. **Probability** The measure of each angle in a triangle is a multiple of 30°. What is the probability that the triangle has at least two congruent angles?

49. In \( \triangle ABC \), \( m \angle B \) is 5° less than \( \frac{1}{2} \) times \( m \angle A \). \( m \angle C \) is 5° less than \( \frac{2}{3} \) times \( m \angle A \). What is \( m \angle A \) in degrees?

**FOCUS ON MATHEMATICAL PRACTICES**

50. **Modeling** Sketch a scalene triangle with a 90° exterior angle.

51. **Justify** A right triangle has an acute angle of 63°. A second right triangle has an acute angle of 27°. How many pairs of congruent angles do the two triangles have? Justify your answer.

52. **Analysis** Explain why a triangle can have, at most, one obtuse angle.

53. **Make a Conjecture** Given that the exterior angle measure of a triangle equals the sum of the two remote interior angle measures, what must be the sum of three exterior angle measures (one at each vertex)? Justify your answer.